Equation 2, $x_2^2-px_2+q=0$, may be solved in a similar manner by changing the sign of px and proceeding as above. The roots being the same in numerical value but opposite in sign.

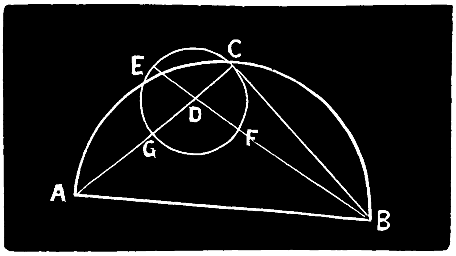
Case II. Equations 3, 4. The figure for Case II differs from that of Case I only in that P is outside of the circle 1; the points CPD being now in the order PCD. $PC=x_1$, $PD=x_2=x_1+p$, $PN.PM=PC.PD$; $r.s=x_1(x_1+p)$, $x_1^2+px_1-rs=0$. Similarly, $x_2^2+px_2-rs=0$.

Equation 4 is solved by changing sign of px and proceeding as with equation 3, since the roots are the opposite of the roots of equation (3).

In Case I if the roots are imaginary, the point P falls within circumference 2, and the graphic method fails.

VI. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Upon AB describe a semi-circle. Let C be the mid-point of the semi-circular arc. Draw AC, BC. Let G be a point on the line AC; on GC describe a circle center D. Through D draw $BFDE$. Then taking BE, BF positive; EB, FB negative, these lines are the roots of a quadratic having $2CD$ for the coefficient of the first power of the unknown quantity, and $\sqrt{(2AB)}$ for the absolute term, the coefficient of the second power of the unknown quantity being taken unity.



Let $AB=\sqrt{2c}$, $GC=2DC=b$.

$BE=BD+DE=BD+DC=\sqrt{(BC^2+DC^2)}+DC=\frac{1}{2}[b+\sqrt{(b^2+4c)}]$.

$BF=BD-DF=BD-DC=-\frac{1}{2}[b-\sqrt{(b^2+4c)}]$. Taking $x^2\pm bx=c$.

then for $x^2+bx=c$, $EB=-\frac{1}{2}[b+\sqrt{(b^2+4c)}]$, $BF=-\frac{1}{2}[b-\sqrt{(b^2+4c)}]$.

For $x^2-bx=c$, $BE=\frac{1}{2}[b+\sqrt{(b^2+4c)}]$, $FB=\frac{1}{2}[b-\sqrt{(b^2+4c)}]$.

If c be negative, the results still hold.

Also solved by G. W. Greenwood, B. A. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill., by use of circle and hyperbola.

219A. Proposed by H. F. MacNEISH, A.B., Assistant in Mathematics. University High School, Chicago, Ill.

Draw a line through a given point which shall divide a given quadrilateral into two equivalent parts; (1) when the point lies in a side of the quadrilateral, (2) when the point is without, (3) within the quadrilateral.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $ABCD$ be the given quadrilateral. Produce DA, CB till they meet in F. Join AC and draw BS parallel to AC, join CS; then triangle SCD=quadri-$ABCD$. Disect SD in H, HF in G, and join CH, CG.

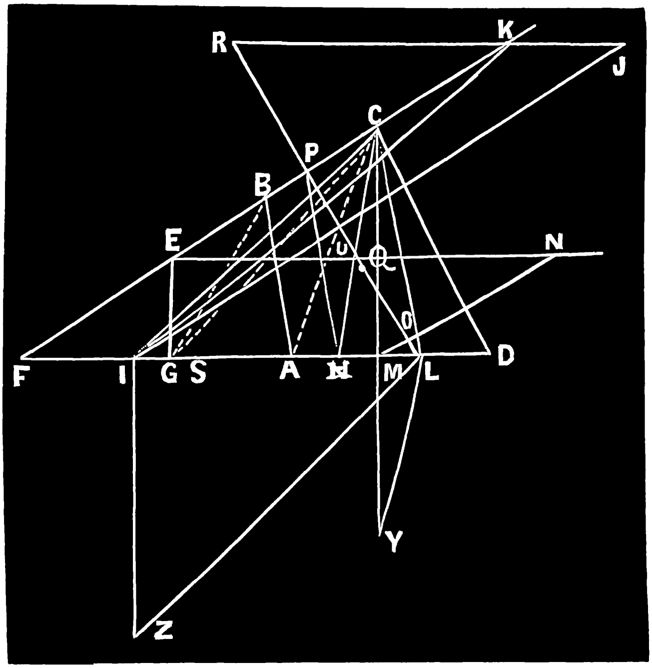
(1). Let P be the point in the side BC. Join PH and draw CL parallel to PH, join PL. Triangle PCL=triangle HCL.

$\therefore PCL+LCD=PCDL=HCL+LCD=HCD=\frac{1}{2}ABCD$.

(2). Let R be the point without the quadrilateral. Draw RK parallel to

FD meeting FC in K. Join KG and draw CI parallel to KG, join KI. Then $ICG=ICK$ and $FCI=FCI$.

$\therefore FCG=FKI$. Draw IJ parallel to FC, then parallelogram $FKJI=FCH$. At the point I draw IZ perpendicular to FD and equal to RK, draw $ZL=RJ$.



Then $RPUL$ is the line required; for $LZ^2-IZ^2=IL^2$ or $VRJ-PRK=IVL=PVJK$.

$\therefore FPL=FKJI=FCH$. Now $SBA=SBC$. $\therefore FBA=FCS$ or $FPL-FBA=FCH-FCS$.

$\therefore ABPL=SCH=\frac{1}{2}ABCD$.

(3). Let Q be the point within the quadrilateral. Draw QE parallel to FD meeting FC in E. Join EG and draw CM parallel to EG, join EM. Then $EGM=EGC$ and $FEM=FCG$. Draw MN parallel to FC; then $FENM=FCH$. At the point M draw MY perpendicular to FD and equal to EQ, draw $YL=QN$. Then $PQOL$ is tne required line, for $LY^2=MY^2+ML^2$ or $QON=PEQ+MOL$.

$\therefore FPL=FENM=FCH$. $\therefore FPL-FBA=FCH-FCS=SCH$.

$\therefore ABPL=SCH=\frac{1}{2}ABCD$.

Also solved by G. W. Greenwood, B. A. (Oxon).

220. Proposed by G. B.M. ZERR, A. M., Ph. D., Parsons, West Va.

Two triangles are circumscribed to a given triangle ABC, having their sides perpendicular to the sides of the given triangle. Prove that the two triangles are equal, and find the area of these triangles.

I. Solution by G. W. GREENWOOD, B. A. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

The perpendiculars to a side of the given triangle at its extremities, which are corresponding sides of the circumscribed triangles, are symmetrical with respect to any point on the perpendicular bisector of that side. Hence the two triangles are symmetric with respect to the circumcenter of the given triangle and are therefore equal. The area of either is equal to

$$\tfrac{1}{2}(c^2\cot B+a^2\cot C+b^2\cot A)+S,$$

where S is the area of the given triangle.